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**On Two Stage Open Shop Scheduling in Which Processing Time, Set Up Time
Associated with Probabilities Including Job Block Criteria**

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Abstract

The present paper is an attempt through heuristic method to obtain the optimal sequence for n jobs two stage open shop problem in which Set up time separated from processing times, each associated with respective probabilities Including job block criteria.. The algorithm developed in this paper is very simple and easy to understand. A numerical illustration to clarify the algorithm is also given.

Keywords: Open shop problem, Equivalent job, Elapsed time, Job block.

Introduction

Now a day, Scheduling has a significant role in the field of manufacturing, management, computer Science, production concern etc. Job sequence and scheduling problem has a wider application in the industrial areas, which minimize the manufacturing cost and time or optimizes effectiveness by selecting the most suitable sequence. The earliest results for flow shop scheduling problem was introduced by Johnson [7] in order to minimize the total idle time of the machines. Further the work was developed by Jackson.J.R.[8], Maggu and Das[9], Harbans[6], Yoshida and Hitomi[13], M.Deel'sAmico[10], D.Rebaine[2], Anup[1], Gupta and Singh [3,5,11], Rastogi and Singh [12], Gupta D; Sharma S.[4] by introducing different parameters such as transportation time ,break down interval, weightage etc. Maggu&Das[9] introduced the concept of equivalent job for a job block in which the priority to one job over another was taken into account. Anup[1] extended the study by associating probabilities with processing time as the processing time are always not exact. Open shop scheduling differ from flow shop in the sense that there are no restrictions placed on the order of the machines i.e. operations can be performed in any order A to B or B to A and not known in the advance. Gupta and Singh [5] studied the $n \times 2$ open shop problem to minimize the total idle time of the machines in which the probabilities associated with processing time including job block criteria. In this paper we have extended the study made by Gupta and Singh [5] by including the concept of set up time separated from processing times, each associated with respective probabilities Including job block criteria Thus the problem in the present paper has wider and practically more applicable and provides suitable results.

An algorithm has been developed to minimize the maximum completion time (makespan). The algorithm is demonstrated through numerical example.

Practical Situation

Many applied and experimental situations occur in day to day working in factories and industrial concern. Open shop scheduling problem has wide engineering application in manufacturing . The practical situation of open shop scheduling may be taken in automobile repair , quality control center , semiconductor manufacturing , satellite communication , class assignment etc. For example in a automobile garage with specialized shop. A car may require the following work: Replacement of exhaust & muffler and alignment of wheels. These two tasks may be carried out in any order on the given machine but it is not possible to perform two tasks simultaneously . The reason is that the exhaust system and alignment are carried out in different shops. One can be performed in any order, even though it is not possible to execute more than one task simultaneously , due to the nature of the job of restriction of resources . Similarly an airplane may be requiring repairs in engine and electrical circuit. These two tasks may be carried out in any order. Sometimes the priority of one job over the other is preferred . It may be because of urgency or demand of its relative important.

Notations

A_i: Processing time of i^{th} job on machine A.

B_i: Processing time of i^{th} job on machine B.

A'_i: Expected Processing time of i^{th} job on machine A.

- B'_i : Expected Processing time of ith job on machine B.
- S_i^A : Expected set up time of ith job on machine A.
- S_i^B : Expected set up time of ith job on machine B.
- P_i : Probability associated to the processing time A_i of ith job on machine A.
- Q_i : Probability associated to the processing time B_i of ith job on machine B.
- G_i : Processing time of ith job on fictitious machine G.
- H_i : Processing time of ith job on fictitious machine H.

Problem Formulation

Let n jobs 1, 2, 3,.....n be processed through two machines A and B. Let A_i and B_i be the processing time of i^{th} job ($i=1,2,3\dots n$) on machine A and B respectively. Let P_i and q_i be the probabilities associated with processing time A_i and B_i respectively $0 \leq P_i \leq 1$, $\sum P_i = 1$, $0 \leq q_i \leq 1$ and $\sum q_i = 1$. Let the setup times S_i^A and S_i^B are being separated from processing time associated with respective probabilities r_i abs s_i on each machine.

Let α be an equivalent job for job block (k,m) in which k is given priority over job m, where k and m are any jobs among the given n jobs.

The mathematical model of the problem can be stated in the matrix form as:

Jobs i	Machine A				Machine B			
	A_i	p_i	S_i^A	s_i	B_i	q_i	S_i^B	r_i
1	A_1	p_1	S_1^A	S_1	B_1	q_1	S_1^B	r_1
2	A_2	p_2	S_2^A	S_2	B_2	q_2	S_2^B	r_2
3	A_3	p_3	S_3^A	S_3	B_3	q_3	S_3^B	r_3
.	.	.	-	-	.	.	-	-
.	.	.	-	-	.	.	-	-
.	.	.	-	-	.	.	-	-
n	A_n	p_n	S_n^A	s_n	B_n	q_n	S_n^B	r_n

Tableau-1

Our objective is to find the optimal schedule of all the jobs which minimize the total elapsed time of each machine, expected processing time may be in the terms of cost.

Algorithm

The heuristic algorithm for the problem discussed here, is as follow as:

Step1: Define two fictitious machines G and H with processing time G_i and H_i as follows :-

- a) $A'_i = A_i \times p_i - S_i^B \times r_i$
- b) $B'_i = B_i \times q_i - S_i^A \times r_i$

Step2: Define expected processing time for the equivalent job $\alpha = (k,m)$ as follows:-

For the order $G \rightarrow H$

- (a) $G'_\alpha = G'_k + G'_m - \min(G'_m, H'_k)$
- (b) $H'_\alpha = H'_k + H'_m - \min(G'_m, H'_k)$

For the order $H \rightarrow G$

- (a) $H'_\alpha = H'_k + H'_m - \min(G'_k, H'_m)$
- (b) $G'_\alpha = G'_k + G'_m - \min(G'_k, H'_m)$

Step3: Represent a new reduced problem with processing time G_i and H_i as per step 2&3

Step5: For the order $G \rightarrow H$

Construct a set S_G of all the processing time G'_i where $G'_i \leq H'_i$ and S'_G of all the processing time G'_i where $G'_i \geq H'_i$.

Step6: Let S_1 denote a sub optimal sequence of jobs corresponding to non decreasing times S_G and similarly a sequence S_2 corresponding to set S'_G .

Step7: The augmented ordered sequence (S_1, S_2) gives optimal or near optimal sequence for processing the jobs on machine A for the given problem.

Step8: For the order $H \rightarrow G$

Construct the set S_H and S'_H of processing times H'_i where $H'_i \leq G'_i$ and of processing times H'_i where $H'_i \geq G'_i$ respectively .

Step9: Let S_2 denote a sub optimal sequence of jobs corresponding to the non decreasing processing times in the set S_H . similarly S'_2 corresponding to S'_H .

Step10: The augmented ordered sequence (S'_1, S'_2) gives the optimal or near optimal sequence for processing the jobs on the machine B for the given problem.

Step11: Prepare in-out tables of sequences (S_1, S_2) and (S'_1, S'_2) in the order $A \rightarrow B$ and $B \rightarrow A$ respectively.

Numerical Illustration: Consider 5 jobs, 2 machines open shop problem with processing time and set up time associated with their respective probabilities as given in the table 2. The jobs 2 and 4 are processed as a group job (1,4).

jobs	Machine A				Machine B			
	A _i	P _i	S _i ^A	s _i	B _i	q _i	S _i ^A	r _i
1	20	0.2	3	0.3	14	0.2	2	0.1
2	18	0.1	4	0.1	17	0.2	5	0.3
3	13	0.1	1	0.2	12	0.1	6	0.2
4	22	0.3	5	0.1	23	0.2	7	0.2
5	10	0.2	4	0.2	18	0.2	3	0.1
6	19	0.1	3	0.2	13	0.1	2	0.1

Tableau-2

Our objective is to find the optimal/ near optimal sequence which minimizes the total elapsed time.

Solution

As per Step1: Define two fictitious machines G and H with their processing times G_i and H_i using: G_i

- (a) $G'_i = A_i \times p_i - S_i^B \times r_i$
- (b) $H'_i = B_i \times q_i - S_i^A \times q_i$

Jobs	G _i	H _i
1	3.8	1.9
2	0.3	3.0
3	0.1	1.0
4	5.2	4.1
5	1.7	2.8
6	1.7	0.7

Tableau-3

As per Step2: For the order G→H

Define processing time for the equivalent job_α = (1, 4) using:

- a) $G'_\alpha = G'_k + G'_m - \min(H'_k, G'_m)$
- b) $G'_\alpha = 3.8 + 5.2 - \min(1.9, 5.2) = 3.8 + 3.3 = 7.1$

$H'_\alpha = H'_k + H'_m - \min(H'_k, G'_m)$

$H'_\alpha = 1.9 + 4.1 - \min(1.9, 5.2) = 1.9 + 2.2 = 4.1$

As per Step3: Reduced processing times G'_i and H'_i are as follows :

Jobs	G' _i	H' _i
α	7.1	4.1
2	0.3	3.0
3	0.1	1.0
5	1.7	2.8
6	1.7	0.7

Tableau-4

As per Step4: Construct a set S_G and S'_G S_G = {0.3, 0.1, 1.7} S'_G = {7.1, 1.7}

As per Step5: S₁ = {3, 2, 5} S'₁ = {6, α}

As per Step6: Augmented ordered sequence = {3, 2, 5, 6, α} i.e. {3, 2, 5, 6, 1, 4}

For the order $H \rightarrow G$: As per step1

Jobs	B'_i	A'_i
1	1.9	3.8
2	3.0	0.3
3	1.0	0.1
4	4.1	5.2
5	2.8	1.7
6	0.7	1.7

Tableau-5

As per Step3: Define processing time for equivalent job $\alpha = (1,4)$

$$\begin{aligned}
 H'_\alpha &= H'_k + H'_m - \min(H'_m, G'_k) \\
 &= 1.9 + 4.1 - \min(4.1, 3.8) \\
 &= 1.9 + 0.3 = 2.2
 \end{aligned}$$

$$\begin{aligned}
 G'_\alpha &= G'_k + G'_m - \min(H'_m, G'_k) \\
 &= 3.8 + 5.2 - \min(4.1, 3.8) = 3.8 + 1.4 = 7.0 = 5.2
 \end{aligned}$$

As per Step7: For the order $H \rightarrow G$, construct the set S_H and S'_H :

Jobs	H'_i	G'_i
α	2.2	5.2
2	3.0	0.3
3	1.0	0.1
5	2.8	1.7
6	0.7	1.7

Tableau-6

$$S_H = \{2.2, 0.7\} \quad S'_H = \{3.0, 1.0, 2.8\}$$

As per Step8: $S_2 = \{6, \alpha\}$ $S'_2 = \{3, 5, 2\}$

As per Step9: Augmented ordered sequence is $= \{6, \alpha, 3, 5, 2\}$ i.e. $\{6, 1, 4, 3, 5, 2\}$

This gives the exact or near optimal sequence of jobs when processed from A to B

Now we calculate the total production times on machine A and B for thesequence $(3, 2, 5, 6, 1, 4)$ and $(6, 1, 4, 3, 5, 2)$ respectively.

As per Step10: In – Out table for the order $A \rightarrow B$

For the order $A \rightarrow B$					
Jobs	Machine A		Machine B		
	In	Out	In	out	
3	0	- 1.3	1.3	- 2.5	
2	1.5	- 3.3	3.7	- 7.3	

5	3.7 - 5.7	8.6 - 12.2
6	6.5 - 8.4	12.5 - 13.8
1	9.0-13.0	14.0 - 16.8
4	13.9 -20.5	20.5 - 25.1

Tableau-7

In – Out table for the order B→A

For the order B→A				
jobs	MachineB		Machine A	
	In	Out	In	Out
6	0.0	- 1.9	1.9	- 3.2
1	2.5	- 6.9	6.9	- 9.7
4	7.8	- 14.4	14.4	-19.0
3	14.9	-16.2	20.4	-21.6
5	16.4	-18.4	22.8	-26.4
2	19.2	- 21.0	26.7	- 30.1

Tableau-8

The total expected elapsed time when the order is from A to B for the sequence (3, 2, 5,6, 1, 4) is 25.1 units and for the sequence (6, 1, 4, 3, 5, 2) is 30.1 units when order is B to A. Hence the optimal sequence of all the jobs which minimize the total elapsed time of each machine is (3, 2, 5, 6, 1, 4,) for the order A→B.

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